

Similarity

Practice Set 1.1

Q. 1. Base of a triangle is 9 and height is 5. Base of another triangle is 10 and height is 6. Find the ratio of areas of these triangles.

Answer : We know that area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$\Rightarrow \text{Area (triangle 1)} = \frac{1}{2} \times 9 \times 5$$

$$= \frac{45}{2}$$

$$\Rightarrow \text{Area (triangle 2)} = \frac{1}{2} \times 10 \times 6$$

$$= 30$$

\therefore The ratio of areas of these triangles will be = $\frac{\text{Area(triangle 1)}}{\text{Area(triangle 2)}}$

$$= \frac{\frac{45}{2}}{30}$$

$$= \frac{45}{2} \times \frac{1}{30}$$

$$= \frac{3}{4}$$

Q. 2. If figure 1.13 $BC \perp AB$, $AD \perp AB$, $BC = 4$, $AD = 8$, then find $\frac{A(\triangle ABC)}{A(\triangle ADB)}$.

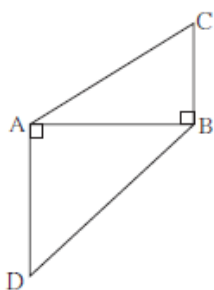


Fig. 1.13

Answer : Here, $\triangle ABC$ and $\triangle ADB$ has common Base.

$$\therefore \frac{\text{Ar}(\triangle ABC)}{\text{Ar}(\triangle ADB)} = \frac{\text{height of } \triangle ABC}{\text{height of } \triangle ADB}$$

(PROPERTY: Areas of triangles with equal bases are proportional to their corresponding heights.)

$$\Rightarrow \frac{\text{Ar}(\triangle ABC)}{\text{Ar}(\triangle ADB)} = \frac{BC}{AD}$$

$$\frac{4}{8}$$

$$\frac{1}{2}$$

Q. 3. In adjoining figure 1.14 seg $PS \perp$ seg RQ , seg $QT \perp$ seg PR . If $RQ = 6$, $PS = 6$ and $PR = 12$, then find QT .

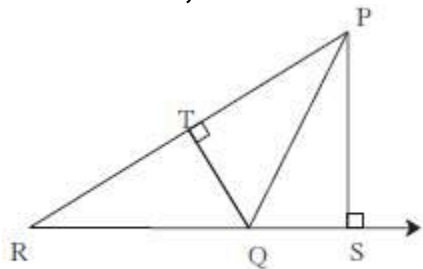


Fig. 1.14

Answer :

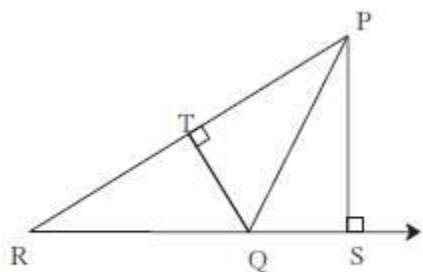


Fig. 1.14

Considering, Area of $(\triangle PQR)$ with base QR

$\Rightarrow PS$ will be the Height

Now, consider the Area of (ΔPQR) with base PR

\Rightarrow QT will be the Height

\therefore The triangle is the same

\Rightarrow The area will be the same irrespective of the base taken.

And we know that area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$\Rightarrow \frac{1}{2} \times QR \times PS$$

$$= \frac{1}{2} \times PR \times QT$$

$$\Rightarrow \frac{1}{2} \times 6 \times 6$$

$$= \frac{1}{2} \times 12 \times QT$$

$$\Rightarrow QT = 3$$

Q. 4. In adjoining figure, $AP \perp BC$, $AD \parallel BC$, then find $A(\Delta ABC) : A(\Delta BCD)$

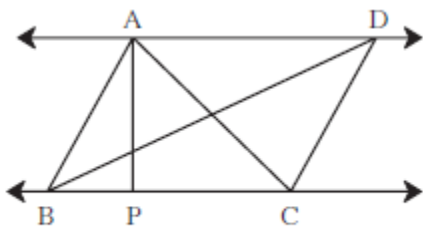
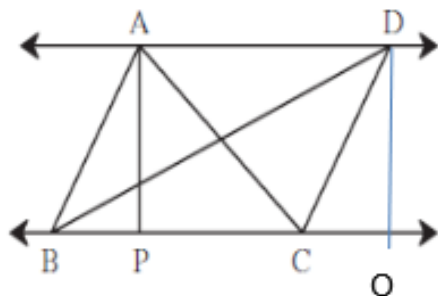


Fig. 1.15

Answer :



We can re-draw the fig. 1.15 (as shown above) where we add DO

Which will be height of $\triangle BCD$.

$$\text{Now, } \frac{A(\triangle ABC)}{A(\triangle BCD)} = \frac{AP}{DO}$$

(PROPERTY: Areas of triangles with equal bases are proportional to their corresponding heights.)

$$\Rightarrow \frac{A(\triangle ABC)}{A(\triangle BCD)} = \frac{AP}{DO}$$

$$\Rightarrow \frac{A(\triangle ABC)}{A(\triangle BCD)} = \frac{1}{1}$$

(\because the distance between the two parallel lines is always equal $\Rightarrow AP = DO$)

$$\Rightarrow \frac{A(\triangle ABC)}{A(\triangle BCD)} = 1:1$$

Q. 5. In adjoining figure $PQ \perp BC$, $AD \perp BC$ then find following ratios.

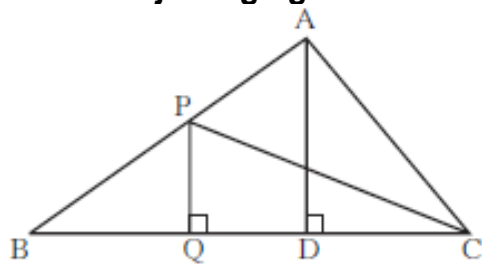


Fig. 1.16

(i) $\frac{A(\triangle PQB)}{A(\triangle PBC)}$

(ii) $\frac{A(\triangle PBC)}{A(\triangle ABC)}$

(iii) $\frac{A(\triangle ABC)}{A(\triangle ADC)}$

(iv) $\frac{A(\triangle ADC)}{A(\triangle PQC)}$

Answer : We know that area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$(i) \frac{A(\triangle PQB)}{A(\triangle PBC)} = \frac{BQ}{BC}$$

(PROPERTY: Areas of triangles with equal heights are proportional to their corresponding bases.)

$$(ii) \frac{A(\triangle PBC)}{A(\triangle ABC)} = \frac{PQ}{AD}$$

(PROPERTY: Areas of triangles with equal bases are proportional to their corresponding heights.)

$$(iii) \frac{A(\triangle ABC)}{A(\triangle ADC)} = \frac{BC}{DC}$$

(PROPERTY: Areas of triangles with equal heights are proportional to their corresponding bases.)

$$(iv) \frac{A(\triangle ADC)}{A(\triangle PQC)} = \frac{\frac{1}{2} \times AD \times DC}{\frac{1}{2} \times PQ \times QC}$$

$$= \frac{AD \times DC}{PQ \times QC}$$

Practice Set 1.2

Q. 1. Given below are some triangles and lengths of line segments. Identify in which figures, ray PM is the bisector of $\angle OPR$.

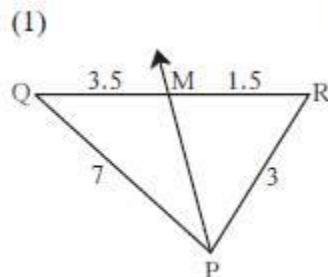


Fig. 1.33

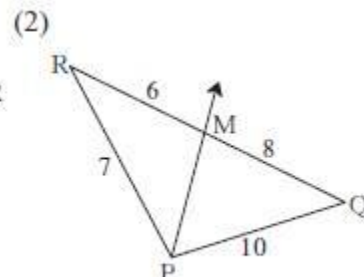


Fig. 1.34

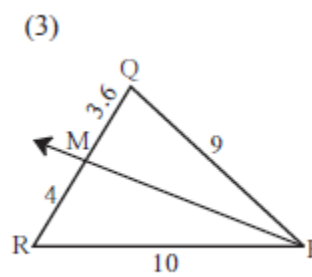


Fig. 1.35

Answer :

Theorem: The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

Therefore, we'll find the ratio for all the triangle. Hence, for

$$(1) \frac{QM}{MR} = \frac{3.5}{1.5}$$

$$= 2.33$$

$$\text{And } \frac{QP}{PR} = \frac{7}{3}$$

$$= 2.33$$

$$\Rightarrow \frac{QM}{MR} = \frac{QP}{PR}$$

\Rightarrow In (1), ray PM is a bisector.

$$(2) \frac{RM}{MQ} = \frac{6}{8}$$

$$= 0.75$$

$$\text{And } \frac{RP}{PQ} = \frac{7}{10}$$

$$= 0.7$$

$$\Rightarrow \frac{RM}{MQ} \neq \frac{RP}{PQ}$$

\Rightarrow In (2), ray PM is not a bisector.

$$(3) \frac{RM}{MQ} = \frac{4}{3.6}$$

$$= 1.1$$

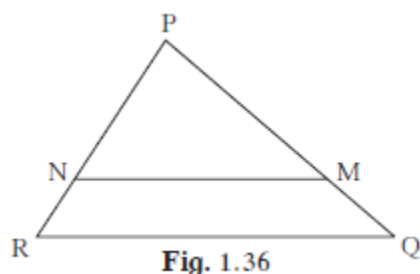
$$\text{And } \frac{RP}{PQ} = \frac{10}{9}$$

$$= 1.11$$

$$\Rightarrow \frac{RM}{MQ} = \frac{RP}{PQ}$$

\Rightarrow In (3), ray PM is a bisector.

Q. 2. In ΔPQR , $PM = 15$, $PQ = 25$, $PR = 20$, $NR = 8$. State whether line NM is parallel to side RQ. Give reason.



Answer : By Converse of basic Proportionality Theorem

(Theorem: If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.)

\Rightarrow If $\frac{PN}{NR} = \frac{PM}{MQ}$, then line NM is parallel to side RQ.

\therefore We'll check if $\frac{PN}{NR} = \frac{PM}{MQ}$.

$$\Rightarrow \frac{PN}{NR} = \frac{PR-NR}{NR}$$

$$= \frac{20-8}{8}$$

$$= \frac{12}{8}$$

$$= \frac{3}{2}$$

And, $\frac{PM}{MQ} = \frac{PM}{PQ-PM}$

$$= \frac{15}{25-15}$$

$$\frac{15}{10}$$

$$= \frac{3}{2}$$

$$\Rightarrow \frac{PN}{NR} = \frac{PM}{MQ} = \frac{3}{2}, \text{ therefore line NM} \parallel \text{side RQ}$$

Q. 3. In ΔMNP , NQ is a bisector of $\angle N$. If $MN = 5$, $PN = 7$ $MQ = 2.5$ then find QP .

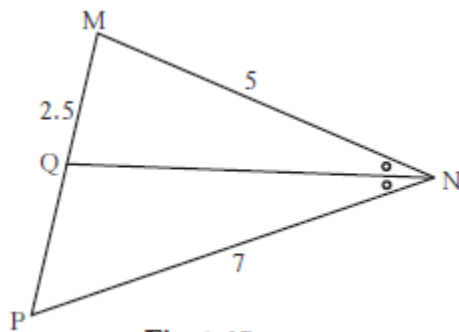


Fig. 1.37

Answer :

Theorem: The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

$$\Rightarrow \frac{MQ}{QP} = \frac{MN}{NP}$$

$$\Rightarrow \frac{2.5}{QP} = \frac{5}{7}$$

$$\Rightarrow QP \times 5 = 2.5 \times 7$$

$$\Rightarrow QP = \frac{2.5 \times 7}{5}$$

$$\Rightarrow QP = 3.5$$

Q. 4. Measures of some angles in the figure are given. Prove that $\frac{AP}{PB} = \frac{AQ}{QC}$

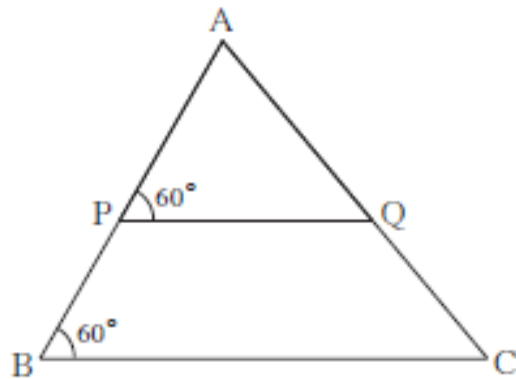


Fig. 1.38

Answer : Here, $PQ \parallel BC$ ($\because \angle APQ \cong \angle ABC$)

(PROPERTY: If a transversal intersects two lines so that corresponding angles are congruent, then the lines are parallel)

\therefore By Basic Proportionality Theorem

(Theorem : If a line parallel to a side of a triangle intersects the remaining sides in two distinct points, then the line divides the sides in the same proportion.)

$$\Rightarrow \frac{AP}{PB} = \frac{AQ}{QC}$$

Q. 5. In trapezium ABCD, side $AB \parallel$ side $PQ \parallel$ side DC , $AP = 15$, $PD = 12$, $QC = 14$, find BQ.

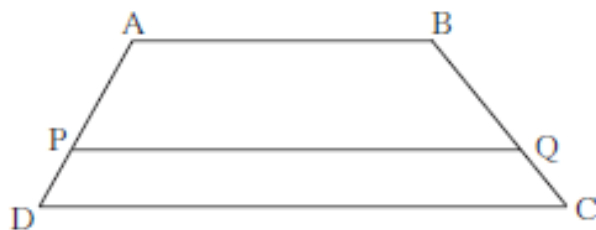


Fig. 1.39

Answer : By Basic Proportionality Theorem

(Theorem : If a line parallel to a side of a triangle intersects the remaining sides in two distinct points, then the line divides the sides in the same proportion.)

$$\Rightarrow \frac{AP}{PD} = \frac{BQ}{QC}$$

$$\Rightarrow \frac{15}{12} = \frac{BQ}{14}$$

$$\Rightarrow BQ = \frac{15 \times 14}{12}$$

$$\Rightarrow BQ = 17.5$$

Q. 6. Find QP using given information in the figure.

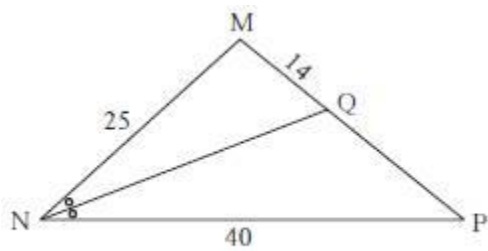


Fig. 1.40

Answer : Theorem: The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

And \because NQ is angle bisector of $\angle N$

$$\Rightarrow \frac{MQ}{QP} = \frac{MN}{NP}$$

$$\Rightarrow \frac{14}{QP} = \frac{25}{40}$$

$$\Rightarrow QP = \frac{14 \times 40}{25}$$

$$\Rightarrow QP = 22.4$$

Q. 7. In figure 1.41, if $AB \parallel CD \parallel FE$ then find x and AE .

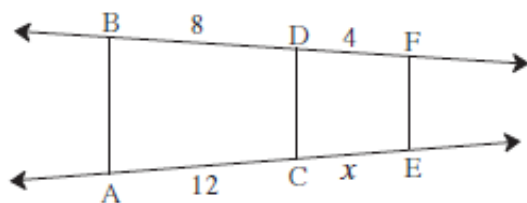


Fig. 1.41

Answer : Theorem: The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines.

$$\Rightarrow \frac{BD}{DF} = \frac{AC}{CE}$$

$$\Rightarrow \frac{8}{4} = \frac{12}{x}$$

$$\Rightarrow x = \frac{12 \times 4}{8}$$

$$\Rightarrow x = 6$$

$$\text{Now, } AE = AC + CE$$

$$= 12 + x$$

$$= 12 + 6$$

$$\Rightarrow AE = 18$$

Q. 8. In $\triangle LMN$, ray MT bisects $\angle LMN$ If $LM = 6$, $MN = 10$, $TN = 8$, then find LT .

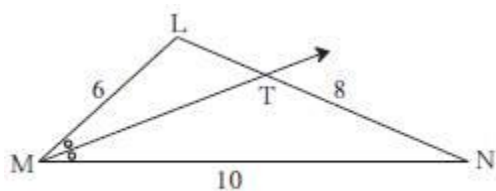


Fig. 1.42

Answer : Theorem: The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

$$\Rightarrow \frac{LT}{TN} = \frac{LM}{MN}$$

$$\Rightarrow LT = \frac{LM \times TN}{MN}$$

$$\Rightarrow LT = \frac{6 \times 8}{10}$$

$$\Rightarrow LT = 4.8$$

Q. 9. In ΔABC , seg BD bisects $\angle ABC$. If $AB = x$, $BC = x + 5$, $AD = x - 2$, $DC = x + 2$, then find the value of x .

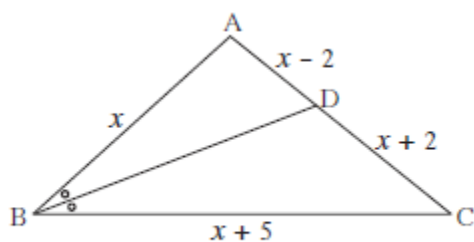


Fig. 1.43

Answer : Theorem: The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

$$\Rightarrow \frac{AD}{DC} = \frac{AB}{BC}$$

$$\Rightarrow \frac{x-2}{x+2} = \frac{x}{x+5}$$

$$\Rightarrow x(x+2) = (x-2)(x+5)$$

$$\Rightarrow x^2 + 2x = x^2 - 2x + 5x - 10$$

$$\Rightarrow x^2 + 2x - x^2 + 2x - 5x + 10 = 0$$

$$\Rightarrow x = 10$$

Q. 10. In the figure 1.44, X is any point in the interior of triangle. Point X is joined to vertices of triangle. Seg PQ \parallel seg DE, seg QR \parallel seg EF. Fill in the blanks to prove that, seg PR \parallel seg DF.

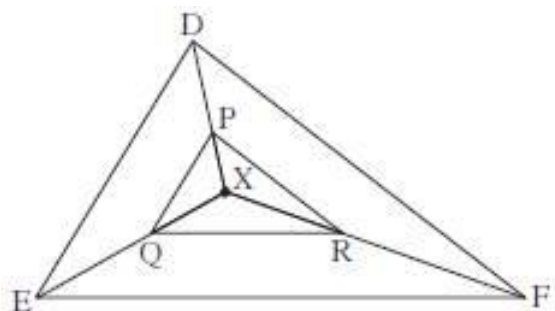


Fig. 1.44

Proof : In $\triangle XDE$, $PQ \parallel DE$ ☐

$$\therefore \frac{XP}{\boxed{}} = \frac{\boxed{}}{QE} \text{(I)}$$

(Basic proportionality theorem)

In $\triangle XDE$, $QR \parallel EF$ ☐

$$\therefore \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}} \text{(II) } \boxed{}$$

$$\therefore \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}} \text{from (I) and (II)}$$

\therefore seg $PR \parallel$ seg DE

(Converse of basic proportionality theorem)

Answer : Proof: In $\triangle XDE$, $PQ \parallel DE$ (Given)

$$\therefore \frac{XP}{XQ} = \frac{PD}{DE} \text{(I)}$$

(Basic proportionality theorem)

In $\triangle XDE$, $QR \parallel EF$ (Given)

$$\therefore \frac{XR}{RF} = \frac{XQ}{QE} \text{(II) (Basic Proportionality Theorem)}$$

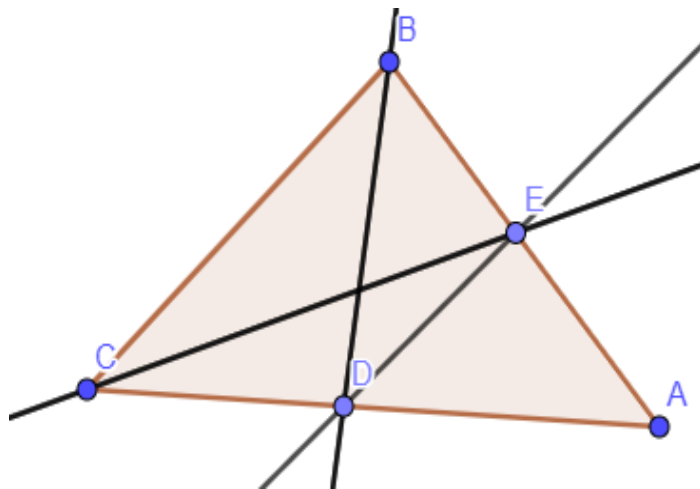
$$\therefore \frac{XP}{PD} = \frac{XR}{RF} \text{ from (I) and (II)}$$

$\therefore \text{seg PR} \parallel \text{Seg DE} \dots\dots\dots$

(converse of basic proportionality theorem)

Q. 11. In $\triangle ABC$, ray BD bisects $\angle ABC$ and ray CE bisects $\angle ACB$. If $\text{seg AB} \cong \text{seg AC}$ then prove that $ED \parallel BC$.

Answer : PROOF:



Theorem: The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

$$\Rightarrow \frac{AD}{DC} = \frac{AB}{CB} \dots\dots(1)$$

$$\text{And } \frac{AE}{EB} = \frac{AC}{CB} \dots\dots(2) \quad (\because \text{BD and CE are angle bisectors of } \angle B \text{ and } \angle C \text{ respectively.})$$

Now, $\because \text{seg AB} \cong \text{seg AC}$

$$\Rightarrow AB = AC$$

$$\Rightarrow \frac{AB}{CB} = \frac{AC}{CB}$$

\Rightarrow R.H.S of (1) & (2) are equal.

\Rightarrow L.H.S of (1) & (2) will be equal.

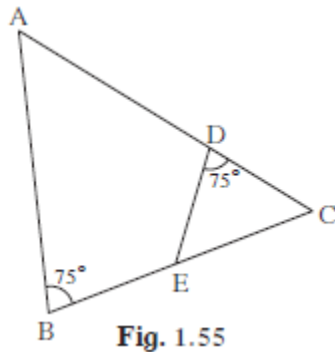
\therefore Equating L.H.S of (1) & (2), we get-

$$\Rightarrow \frac{AD}{DC} = \frac{AE}{EB}$$

$\Rightarrow ED \parallel BC$ (By converse basic proportionality theorem)

Practice Set 1.3

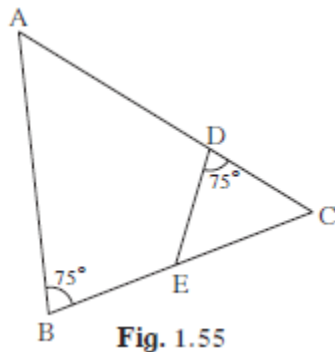
Q. 1. In figure 1.55, $\angle ABC = 75^\circ$, $\angle EDC = 75^\circ$ state which two triangles are similar and by which test? Also write the similarity of these two triangles by a proper one to one correspondence.



Answer : With one- to-one correspondence $ABC \leftrightarrow EDC$

$$\because \angle ABC \cong \angle EDC = 75^\circ$$

$\angle ACB \cong \angle ECD$ (Is common in both the triangles ABC and EDC)



$\Rightarrow \triangle ABC \sim \triangle EDC$ (By AA Test)

Q. 2. Are the triangles in figure 1.56 similar? If yes, by which test?

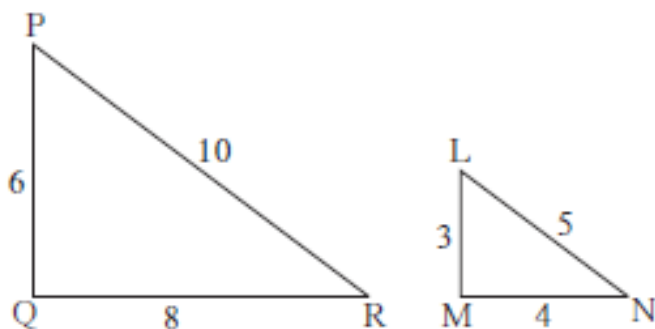


Fig. 1.56

Answer : In ΔPQR and ΔLMN

$$\frac{PQ}{LM} = \frac{6}{3} = 2$$

$$\text{And } \frac{QR}{MN} = \frac{8}{4} = 2$$

$$\text{And } \frac{PR}{LN} = \frac{10}{5} = 2$$

$$\Rightarrow \frac{PQ}{LM} = \frac{QR}{MN} = \frac{PR}{LN} = 2$$

$\Rightarrow \Delta PQR \sim \Delta LMN$ (By SSS Similarity Test)

Q. 3. As shown in figure 1.57, two poles of height 8 m and 4 m are perpendicular to the ground. If the length of shadow of smaller pole due to sunlight is 6 m then how long will be the shadow of the bigger pole at the same time?

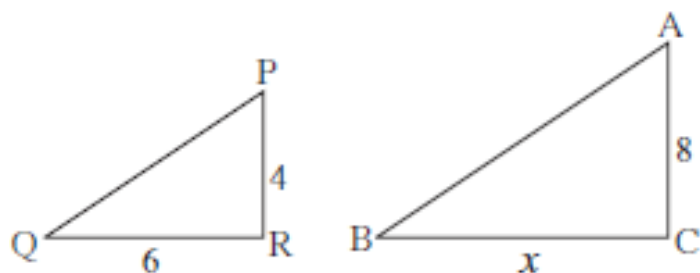


Fig. 1.57

Answer : \because The shadows are measured at the same time

⇒ Angle of elevation will be equal for both the pole

⇒ $\Delta PQR \sim \Delta ABC$ (By AA Test)

$$\Rightarrow \frac{PR}{AC} = \frac{QR}{BC}$$

$$\Rightarrow BC = \frac{QR \times AC}{PR}$$

$$\Rightarrow x = \frac{6 \times 8}{4}$$

$$\Rightarrow x = 12 \text{ m}$$

Q. 4. In ΔABC , $AP \perp BC$, $BQ \perp AC$ B- P-C, A-Q - C then prove that, $\Delta CPA \sim \Delta CQB$.
If $AP = 7$, $BQ = 8$, $BC = 12$ then find AC.

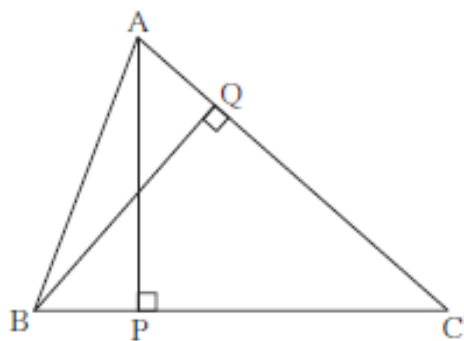


Fig. 1.58

Answer : From fig.

⇒ $\angle APC \cong \angle BQC$ ($\because AP \perp BC$ and $BQ \perp AC$)

⇒ Also, $\angle ACP \cong \angle BCQ$ (Common)

⇒ $\Delta CPA \sim \Delta CQB$ (By AA Test)

$$\Rightarrow \frac{AP}{BQ} = \frac{AC}{BC}$$

$$\Rightarrow AC = \frac{AP \times BC}{BQ}$$

$$\Rightarrow AC = \frac{7 \times 12}{8}$$

$$\Rightarrow AC = 10.5$$

Q. 5. Given : In trapezium PQRS, side PQ || side SR, AR = 5AP, AS = 5AQ then prove that, SR = 5PQ

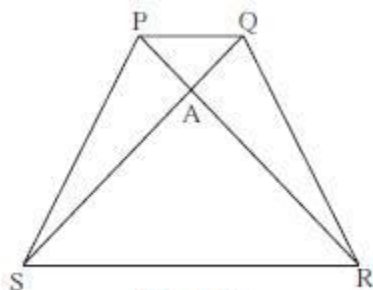


Fig. 1.59

Answer : Given that, AR = 5AP and AS = 5AQ

$$\Rightarrow \frac{AR}{AP} = 5 \dots\dots\dots(1)$$

$$\text{And } \frac{AS}{AQ} = 5 \dots\dots\dots(2)$$

$$\Rightarrow \frac{AR}{AP} = \frac{AS}{AQ}$$

And, $\angle SAR \cong \angle QAP$ (Opposite angles)

$\Rightarrow \Delta SAR \sim \Delta QAP$ (SAS Test of similarity)

$$\Rightarrow \frac{AS}{AQ} = \frac{AR}{AP} = \frac{SR}{QP} \text{ (corresponding sides are proportional)}$$

$$\text{But, } \frac{AS}{AQ} = \frac{AR}{AP} = 5$$

$$\Rightarrow \frac{SR}{QP} = 5$$

$$\Rightarrow SR = 5PQ$$

Q. 6. In trapezium ABCD, (Figure 1.60) side AB || side DC, diagonals AC and BD intersect in point O. If AB = 20, DC = 6, OB = 15 then find OD.

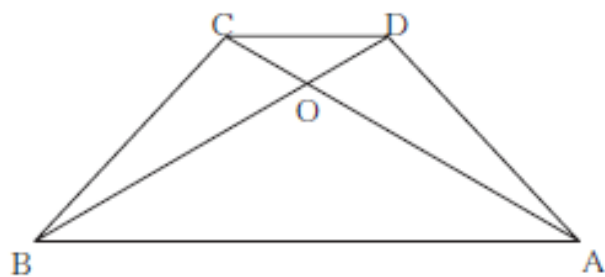


Fig. 1.60

Answer : In $\triangle AOB$ and $\triangle COD$

$\Rightarrow \angle AOB \cong \angle COD$ (opposite angles)

$\Rightarrow \angle CDO \cong \angle ABO$ (Alternate angles $\because AB \parallel DC$)

$\Rightarrow \triangle AOB \sim \triangle COD$ (By AA Test)

$\Rightarrow \frac{AB}{DC} = \frac{OB}{OD}$ (corresponding sides are proportional)

$\Rightarrow OD = \frac{OB \times DC}{AB}$

$\Rightarrow OD = \frac{15 \times 6}{20}$

$\Rightarrow OD = 4.5$

Q. 7. ABCD is a parallelogram point E is on side BC. Line DE intersects ray AB in point T. Prove that $DE \times BE = CE \times TE$.

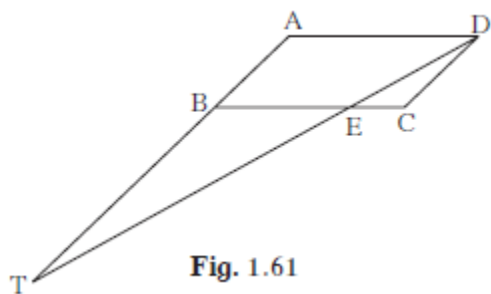


Fig. 1.61

Answer : In $\triangle CED$ and $\triangle BET$

$\Rightarrow \angle CED \cong \angle BET$ (opposite angles)

$\Rightarrow \angle CDE \cong \angle BTE$ (Alternate angles)

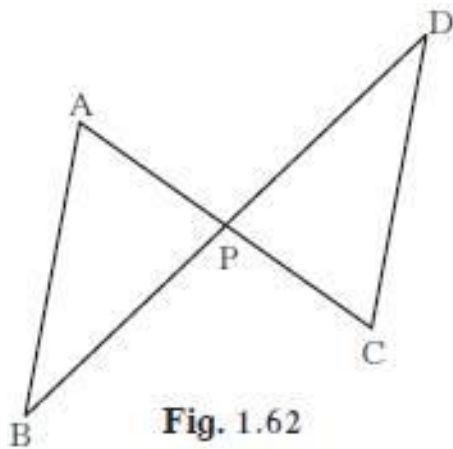
($\because AB \parallel DC \Rightarrow BT \parallel DC$, as BT is extension to AB)

$\Rightarrow \triangle CED \sim \triangle BET$ (By AA Test)

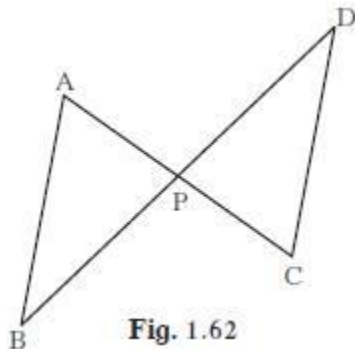
$\Rightarrow \frac{CE}{DE} = \frac{BE}{TE}$ (corresponding sides are proportional)

$\Rightarrow DE \times BE = CE \times TE$

Q. 8. In the figure, seg AC and seg BD intersect each other in point P and $\frac{AP}{CP} = \frac{BP}{DP}$. Prove that, $\triangle ABP \sim \triangle CDP$



Answer :



In $\triangle APB$ & $\triangle CPD$

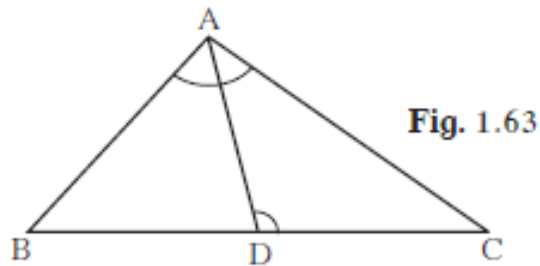
$$\Rightarrow \frac{AP}{CP} = \frac{BP}{DP} \dots\dots(\text{Given})$$

And, $\angle APB = \angle DPC$ (vertically opposite angles)

$\Rightarrow \Delta APB \sim \Delta CPD$ (By SAS Test)

Q. 9. In the figure, in ΔABC , point D on side BC is such that, $\angle BAC = \angle ADC$.

Prove that, $CA^2 = CB \times CD$



Answer : In ΔBAC & ΔADC

$\Rightarrow \angle BAC \cong \angle ADC \dots\dots(\text{Given})$

And, $\angle ACB \cong \angle DCA \dots\dots(\text{common})$

$\Rightarrow \Delta BAC \sim \Delta ADC$ (By AA Test)

$$\Rightarrow \frac{CA}{CD} = \frac{CB}{CA} \text{ (corresponding sides are proportional)}$$

$$\Rightarrow CA^2 = CB \times CD$$

Practice Set 1.4

Q. 1. The ratio of corresponding sides of similar triangles is 3 : 5; then find the ratio of their areas.

Answer : Theorem: When two triangles are similar, the ratio of areas of those triangles is equal to the ratio of the squares of their corresponding sides.

$$\Rightarrow \text{Ratio of areas} = 3^2:5^2$$

$$\Rightarrow \text{Ratio of areas} = 9 : 25$$

Q. 2. If $\Delta ABC \sim \Delta PQR$ and $AB:PQ = 2:3$, then fill in the blanks.

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{\square} = \frac{2^2}{3^2} = \frac{\square}{\square}$$

Answer : $\because \Delta ABC \sim \Delta PQR$ and $AB:PQ = 2:3$

$$\Rightarrow \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{2^2}{3^2} = \frac{4}{9}$$

Q. 3. If $\Delta ABC \sim \Delta PQR$, $A(\Delta ABC) = 80$, $A(\Delta PQR) = 125$, then fill in the blanks.

$$\frac{A(\Delta ABC)}{A(\Delta \dots)} = \frac{80}{125} \quad \therefore \frac{AB}{PQ} = \frac{\square}{\square}$$

Answer : $\because \Delta ABC \sim \Delta PQR$

$$\Rightarrow \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{80}{125} \quad (\because A(\Delta PQR) = 125 \text{ is given})$$

$$\Rightarrow \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$$

$$\Rightarrow \frac{AB}{PQ} = \sqrt{\frac{A(\Delta ABC)}{A(\Delta PQR)}}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{4}{5}$$

Q. 4. $\Delta LMN \sim \Delta PQR$, $9 \times A(\Delta PQR) = 16 \times A(\Delta LMN)$. If $QR = 20$ then find MN .

Answer : $\because \Delta ABC \sim \Delta PQR$

\Rightarrow Given that, $9 \times A(\Delta ABC) = 16 \times A(\Delta PQR)$

$$\Rightarrow \frac{A(\Delta PQR)}{A(\Delta LMN)} = \frac{16}{9}$$

$$\text{And, } \frac{A(\Delta PQR)}{A(\Delta LMN)} = \frac{QR^2}{MN^2}$$

$$\Rightarrow \frac{QR^2}{MN^2} = \frac{16}{9}$$

$$\Rightarrow \frac{20^2}{MN^2} = \frac{16}{9}$$

$$\Rightarrow MN^2 = \frac{400 \times 9}{16}$$

$$\Rightarrow MN = 15$$

Q. 5. Areas of two similar triangles are 225 sq.cm & 81 sq.cm. If a side of the smaller triangle is 12 cm, then find corresponding side of the bigger triangle.

Answer : Let area of one(bigger) triangle be 'A', other(smaller) triangle be 'B', corresponding side of smaller triangle be 'a' and bigger triangle be 'b'.

$$\Rightarrow \frac{A}{B} = \frac{b^2}{a^2} \text{ (By theorem)}$$

And a = 12cm, A = 225 sq.cm, B = 81 sq.cm(Given)

$$\Rightarrow \frac{225}{81} = \frac{b^2}{12^2}$$

$$\Rightarrow b^2 = \frac{225 \times 144}{81}$$

$$\Rightarrow b = \sqrt{400}$$

$$\Rightarrow b = 20 \text{ cm}$$

Q. 6. ΔABC and ΔDEF are equilateral triangles. If $A(\Delta ABC) : A(\Delta DEF) = 1 : 2$ and $AB = 4$, find DE .

Answer : We know that, all the angles of an equilateral triangles are equal, i.e., 60° .

$\Rightarrow \Delta ABC \sim \Delta DEF$ (By AAA Similarity Test)

$$\Rightarrow \frac{A(\Delta ABC)}{A(\Delta DEF)} = \frac{AB^2}{DE^2}$$

$$\text{And, } \frac{A(\Delta ABC)}{A(\Delta DEF)} = \frac{1}{2} \text{ (Given)}$$

$$\Rightarrow \frac{AB^2}{DE^2} = \frac{1}{2}$$

$$\Rightarrow DE^2 = 2 \times 4^2 (\because AB = 4)$$

$$\Rightarrow DE = \sqrt{32}$$

$$\Rightarrow DE = 4\sqrt{2}$$

Q. 7. In figure 1.66, seg PQ || seg DE, A(Δ PQF) = 20 units, PF = 2 DP, then find A(DPQE) by completing the following activity.

A(Δ PQF) = 20 units, PF = 2 DP, Let us assume DP = x. \therefore PF = 2x

$$DF = DP + \square = \square + \square = 3x$$

In Δ FDE and Δ FPQ,

$\angle FDE \cong \angle$ corresponding angles

$\angle FED \cong \angle$ corresponding angles

$\therefore \Delta FDE \sim \Delta FPQ$ AA test

$$\therefore \frac{A(\Delta FDE)}{A(\Delta FPQ)} = \frac{\square}{\square} = \frac{(3x)^2}{(2x)^2} = \frac{9}{4}$$

$$A(\Delta FDE) = \frac{9}{4} A(\Delta FPQ) = \frac{9}{4} \times \square = \square$$

$$A(\square DPQE) = A(\Delta FDE) - A(\Delta FPQ)$$

$$= \square - \square$$

$$= \square$$

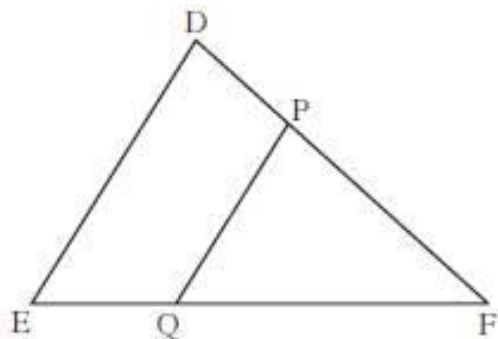


Fig. 1.66

Answer : $A(\Delta PQF) = 20\text{units}$, $PF = 2DP$, Let us assume $DP = x$, $\therefore PF = 2x$

$$\Rightarrow DF = DP + PF = x + 2x = 3x$$

In ΔFDE & ΔFPQ

$\angle FDE \cong \angle FPQ$ (Corresponding angles)

$\angle FED \cong \angle FQP$ (Corresponding angles)

$\therefore \Delta FDE \sim \Delta FPQ$ (AA Test)

$$\therefore \frac{A(\Delta FDE)}{A(\Delta FPQ)} = \frac{DF^2}{PF^2} = \frac{(3x)^2}{(2x)^2} = \frac{9}{4}$$

$$A(\Delta FDE) = \frac{9}{4} A(\Delta FPQ) = \frac{9}{4} \times 20 = 45$$

$$A(\square DPQE) = A(\Delta FDE) - A(\Delta FPQ)$$

$$= 45 - 20$$

$$= 25 \text{ sq. unit.}$$

Problem Set 1

Q. 1. A. Select the appropriate alternative.

In ΔABC and ΔPQR , in a one to one correspondence $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then

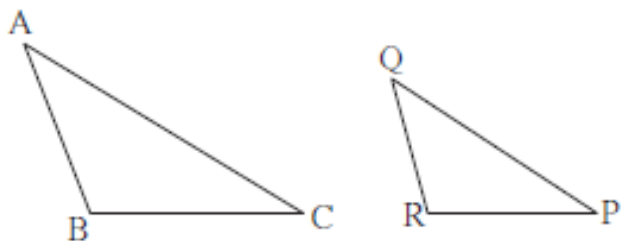


Fig. 1.67

A. $\Delta PQR \sim \Delta ABC$

B. $\Delta PQR \sim \Delta CAB$

C. $\Delta CBA \sim \Delta PQR$

D. $\Delta BCA \sim \Delta PQR$

Answer : $\because \frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$

$\Rightarrow \Delta CAB \sim \Delta PQR$

(A) doesn't match the solution.

(C) doesn't match the solution.

(D) doesn't match the solution.

Q. 1. B. Select the appropriate alternative.

If in ΔDEF and ΔPQR , $\angle D \cong \angle Q$, $\angle R \cong \angle E$ then which of the following statements is false?

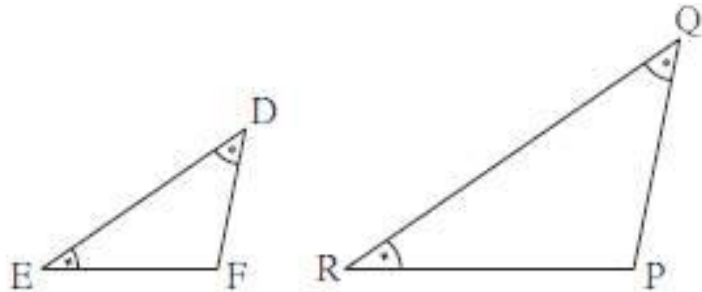


Fig. 1.68

A. $\frac{EF}{PR} = \frac{DF}{PQ}$

B. $\frac{DE}{PQ} = \frac{EF}{RP}$

C. $\frac{DE}{QR} = \frac{DF}{PQ}$

D. $\frac{EF}{RP} = \frac{DE}{QR}$

Answer : In ΔDEF & ΔPQR

$\angle D \cong \angle Q$ and $\angle R \cong \angle E$ (Given)

$\Rightarrow \Delta DEF \sim \Delta PQR$

$$\Rightarrow \frac{DE}{PQ} = \frac{EF}{QR} = \frac{FD}{RP} \text{ (corresponding sides are proportional)}$$

(A) Is matching the solution, hence can't be false.

(C) Is matching the solution, hence can't be false.

(D) Is matching the solution, hence can't be false.

Q. 1. C. Select the appropriate alternative.

In Δ and ΔDEF $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$ then which of the statements regarding the two triangles is true?

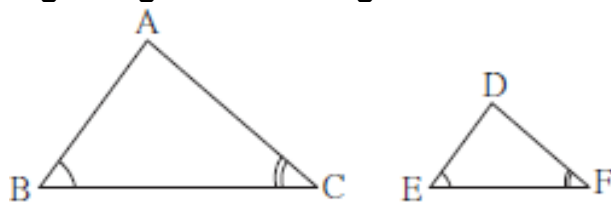


Fig. 1.69

A. The triangles are not congruent and not similar

B. The triangles are similar but not congruent.

C. The triangles are congruent and similar.

D. None of the statements above is true.

Answer : In ΔABC & ΔDEF

$\angle B \cong \angle E$ and $\angle C \cong \angle F$ (Given)

$\Rightarrow \Delta ABC \sim \Delta DEF$ (By AA Test)

\Rightarrow The triangles are similar.

And, $\Delta ABC \cong \Delta DEF$, if $AB = DE$.

But, given that - $AB = 3DE$.

\Rightarrow The triangles are not congruent.

(A) doesn't match the solution.

(C) doesn't match the solution.

(D) doesn't match the solution.

Q. 1. D. Select the appropriate alternative.

ΔABC and ΔDEF are equilateral triangles, $A(\Delta ABC) : A(\Delta DEF) = 1 : 2$. If $AB = 4$ then what is length of DE ?

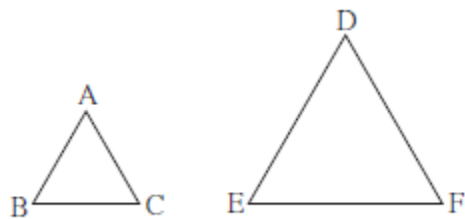


Fig. 1.70

A. $2\sqrt{2}$

B. 4

C. 8

D. $4\sqrt{2}$

Answer : Solution: We know that, all the angles of an equilateral triangles are equal, i.e., 60° .

$\Rightarrow \Delta ABC \sim \Delta DEF$ (By AAA Similarity Test)

$$\Rightarrow \frac{A(\Delta ABC)}{A(\Delta DEF)} = \frac{AB^2}{DE^2}$$

And, $\frac{A(\Delta ABC)}{A(\Delta DEF)} = \frac{1}{2}$ (Given)

$$\Rightarrow \frac{AB^2}{DE^2} = \frac{1}{2}$$

$$\Rightarrow DE^2 = 2 \times 4^2 (\because AB = 4)$$

$$\Rightarrow DE = \sqrt{32}$$

$$\Rightarrow DE = 4\sqrt{2}$$

(A) doesn't match the solution.

(B) doesn't match the solution.

(C) doesn't match the solution.

Q. 1. E. Select the appropriate alternative.

In figure 1.71, seg XY || seg BC, then which of the following statements is true?

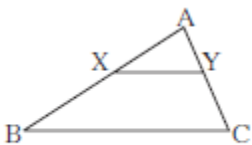


Fig. 1.71

A. $\frac{AB}{AC} = \frac{AX}{AY}$

B. $\frac{AX}{XB} = \frac{AY}{AC}$

C. $\frac{AX}{YC} = \frac{AY}{XB}$

D. $\frac{AB}{YC} = \frac{AC}{XB}$

Answer : \because segXY || segBC

$\Rightarrow \angle AXY \cong \angle ABC$

And, $\angle XAY \cong \angle BAC$ (Common)

$\Rightarrow \Delta AXY \sim \Delta ABC$ (By AA Test)

$\Rightarrow \frac{AX}{AB} = \frac{AY}{AC} = \frac{XY}{BC}$ (corresponding sides are proportional)

$\Rightarrow \frac{AB}{AC} = \frac{AX}{AY}$

(B) doesn't match the solution.

(C) doesn't match the solution.

(D) doesn't match the solution.

Q. 2. In ΔABC , B - D - C and $BD = 7$, $BC = 20$ then find following ratios.

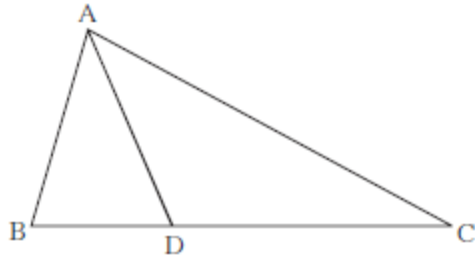


Fig. 1.72

$$(1) \frac{A(\triangle ABD)}{A(\triangle ADC)}$$

$$(2) \frac{A(\triangle ABD)}{A(\triangle ABC)}$$

$$(3) \frac{A(\triangle ADC)}{A(\triangle ABC)}$$

Answer : Theorem: When two triangles are similar, the ratio of areas of those triangles is equal to the ratio of the squares of their corresponding sides.

$$(1) \frac{A(\triangle ABD)}{A(\triangle ADC)} = \frac{BD^2}{DC^2}$$

$$= \frac{BD^2}{(BC-BD)^2}$$

$$= \frac{7^2}{(20-7)^2}$$

$$= \frac{7^2}{13^2}$$

$$(2) \frac{A(\triangle ABD)}{A(\triangle ABC)} = \frac{BD^2}{BC^2}$$

$$= \frac{BD^2}{BC^2}$$

$$= \frac{7^2}{20^2}$$

$$(3) \frac{A(\triangle ADC)}{A(\triangle ABC)} = \frac{DC^2}{BC^2}$$

$$= \frac{(BC-BD)^2}{BC^2}$$

$$= \frac{(20-7)^2}{20^2}$$

$$= \frac{13^2}{20^2}$$

Q. 3. Ratio of areas of two triangles with equal heights is 2 : 3. If base of the smaller triangle is 6 cm then what is the corresponding base of the bigger triangle?

Answer : (PROPERTY: Areas of triangles with equal heights are proportional to their corresponding bases.)

$$\Rightarrow \frac{A(\text{smaller triangle})}{A(\text{bigger triangle})} = \frac{\text{base}(\text{smaller triangle})}{\text{base}(\text{bigger triangle})}$$

$$\Rightarrow \frac{2}{3} = \frac{6}{\text{base}(\text{bigger triangle})}$$

$$\Rightarrow \text{Base (bigger triangle)} = 9 \text{ cm}$$

Q. 4. In figure 1.73, $\angle ABC = \angle DCB = 90^\circ$ $AB = 6$, $DC = 8$ then $\frac{A(\triangle ABC)}{A(\triangle DCB)}$?

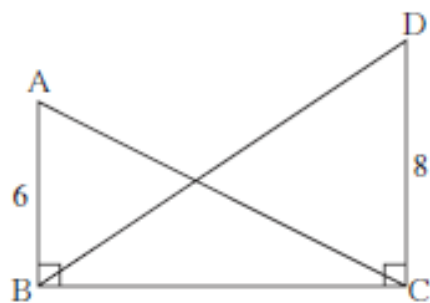


Fig. 1.73

Answer : We know that, Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

$$\Rightarrow \frac{A(\triangle ABC)}{A(\triangle DCB)} = \frac{\frac{1}{2} \times BC \times AB}{\frac{1}{2} \times BC \times DC}$$

$$= \frac{AB}{DC}$$

$$\frac{6}{8}$$

$$\frac{3}{4}$$

Q. 5. In figure 1.74, $PM = 10$ cm $A(\Delta PQS) = 100$ sq.cm $A(\Delta QRS) = 110$ sq.cm then find NR.

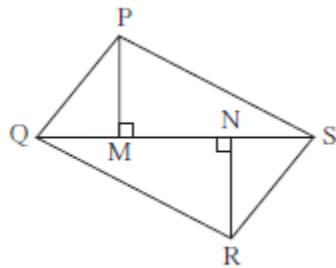


Fig. 1.74

Answer : We know that, Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

$$\Rightarrow \frac{A(\Delta PQS)}{A(\Delta QRS)} = \frac{\frac{1}{2} \times QS \times PM}{\frac{1}{2} \times QS \times NR}$$

$$\Rightarrow \frac{100}{110} = \frac{PM}{NR}$$

$$\Rightarrow \frac{100}{110} = \frac{10}{NR}$$

$$\Rightarrow NR = 11 \text{ cm}$$

Q. 6. $\Delta MNT \sim \Delta QRS$. Length of altitude drawn from point T is 5 and length of altitude drawn from point S is 9. Find the ratio $\frac{A(\Delta MNT)}{A(\Delta QRS)}$.

$$\text{Answer : } \frac{A(\Delta MNT)}{A(\Delta QRS)} = \frac{(\text{altitude from T})^2}{(\text{altitude from S})^2}$$

$$= \frac{5^2}{9^2}$$

$$= \frac{25}{81}$$

Q. 7. In figure 1.75, A – D – C and B – E – C seg DE || side AB If AD = 5, DC = 3, BC = 6.4 then find BE.

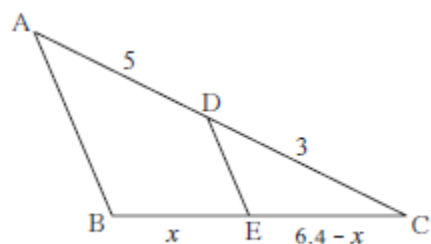


Fig. 1.75

Answer : By Basic Proportionality Theorem-

$$\Rightarrow \frac{CD}{DA} = \frac{CE}{EB}$$

$$\Rightarrow \frac{3}{5} = \frac{6.4-x}{x}$$

$$\Rightarrow 3x = 32 - 5x$$

$$\Rightarrow 8x = 32$$

$$\Rightarrow x = 4 = BE$$

Q. 8. In the figure 1.76, seg PA, seg QB, seg RC and seg SD are perpendicular to line AD.

AB = 60, BC = 70, CD = 80, PS = 280 then find PQ, QR and RS.

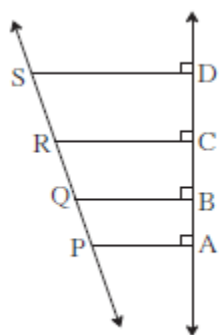


Fig. 1.76

Answer : (PROPERTY: If line AX || line BY || line CZ and line l and line m are their transversals then)

$$\frac{AB}{BC} = \frac{XY}{YZ}$$

$$\Rightarrow \frac{AB}{BC} = \frac{PQ}{QR}$$

$$\Rightarrow \frac{60}{70} = \frac{PQ}{QR}$$

$$\Rightarrow \frac{PQ}{QR} = \frac{6}{7}$$

$$\Rightarrow PQ = \frac{6}{7}QR \quad [1]$$

$$\text{And } \frac{BC}{CD} = \frac{QR}{RS}$$

$$\Rightarrow \frac{70}{80} = \frac{QR}{RS}$$

$$\Rightarrow \frac{QR}{RS} = \frac{7}{8}$$

$$\Rightarrow RS = \frac{8}{7}QR \quad [2]$$

And, PS = 280

$$\Rightarrow PQ + QR + RS = 280 \dots\dots(3)$$

From [1] and [2], we have

$$\Rightarrow \frac{6}{7}QR + QR + \frac{8}{7}QR = 280$$

$$\Rightarrow \frac{14}{7}QR + QR = 280$$

$$\Rightarrow 2QR + QR = 280$$

$$\Rightarrow 3QR = 280$$

$$\Rightarrow QR = \frac{280}{3}$$

$$PQ = \frac{6}{7}QR$$

$$\Rightarrow PQ = \frac{6}{7} \times \frac{280}{3} = 80$$

From [1],

$$RS = \frac{8}{7} \times \frac{280}{3} = \frac{320}{3}$$

From [2]

Q. 9. In ΔPQR seg PM is a median. Angle bisectors of $\angle PMQ$ and $\angle PMR$ intersect side PQ and side PR in points X and Y respectively. Prove that $XY \parallel QR$.

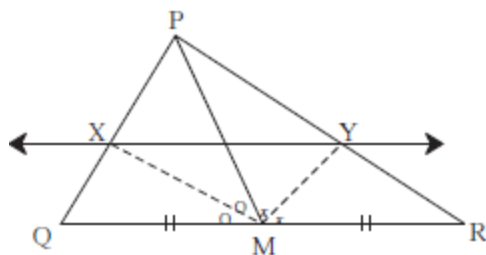


Fig. 1.77

Complete the proof by filling in the boxes. In ΔPMQ , ray MX is bisector of $\angle PMQ$.

$$\therefore \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

..... (I) theorem of angle bisector.

In ΔPMR , ray MY is bisector of $\angle PMR$.

$$\therefore \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

..... (II) Theorem of angle bisector.

But $\frac{MP}{MQ} = \frac{MP}{MR}$ M is the midpoint QR, hence $MQ = MR$.

$$\therefore \frac{PX}{XQ} = \frac{PY}{YR}$$

$\therefore XY \parallel QR$ converse of basic proportionality theorem.

Answer : $\therefore \frac{PM}{MQ} = \frac{PX}{XQ}$ (I) theorem of angle bisector.

And

$$\therefore \frac{PM}{MR} = \frac{PY}{YR} \dots\dots\dots \text{(II) Theorem of angle bisector.}$$

Q. 10. In fig 1.78, bisectors of $\angle B$ and $\angle C$ of ΔABC intersect each other in point X. Line AX intersects side BC in point Y. $AB = 5$, $AC = 4$, $BC = 6$ then find $\frac{AX}{XY}$.

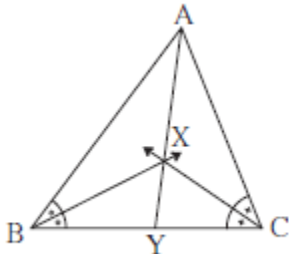


Fig. 1.78

Answer : By Bisector Theorem-

$$\Rightarrow \frac{AX}{XY} = \frac{AB}{BY} \dots\dots(1)$$

$$\Rightarrow \text{And, } \frac{AX}{XY} = \frac{AC}{CY} \dots\dots(2)$$

Equating (1) & (2), we get-

$$\Rightarrow \frac{AB}{BY} = \frac{AC}{CY}$$

$$\Rightarrow \frac{AB}{AC} = \frac{BY}{CY}$$

$$\Rightarrow \frac{AB + AC}{AC} = \frac{BY + CY}{CY}$$

$$= \frac{BC}{CY}$$

$$\Rightarrow \frac{5 + 4}{4} = \frac{6}{CY}$$

$$\Rightarrow CY = \frac{24}{9}$$

$$\Rightarrow CY = \frac{8}{3}$$

$$\text{Now, } \frac{AX}{XY} = \frac{AC}{CY}$$

$$\Rightarrow \frac{AX}{XY} = \frac{4}{\frac{8}{3}}$$

$$\Rightarrow \frac{AX}{XY} = \frac{3}{2}$$

Q. 11. In $\square ABCD$, seg $AD \parallel$ seg BC . Diagonal AC and diagonal BD intersect each other in point P . Then show that $\frac{AP}{PD} = \frac{PC}{BP}$

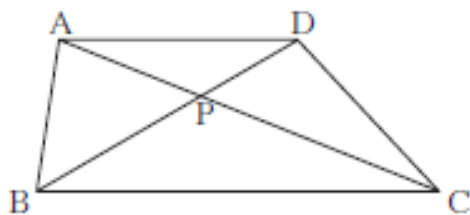


Fig. 1.79

Answer : In $\triangle APD$ and $\triangle CPB$

$\Rightarrow \angle APD \cong \angle CPB$ (opposite angles)

$\Rightarrow \angle ADP \cong \angle PBC$ (Alternate angles $\because AD \parallel BC$)

$\Rightarrow \triangle APD \sim \triangle CPB$ (By AA Test)

$\Rightarrow \frac{AP}{PC} = \frac{PD}{BP}$ (corresponding sides are proportional)

$$\Rightarrow \frac{AP}{PD} = \frac{PC}{BP}$$

Q. 12. In fig 1.80, $XY \parallel$ seg AC . If $2AX = 3BX$ and $XY = 9$. Complete the activity to find the value of AC .

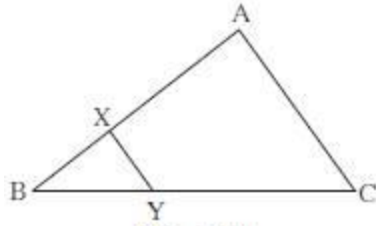


Fig. 1.80

Activity : $2AX = 3BX \therefore \frac{AX}{BX} = \frac{\boxed{}}{\boxed{}}$

$$\frac{AX + BX}{BX} = \frac{\boxed{} + \boxed{}}{\boxed{}} \text{ by componendo.}$$

$$\frac{AB}{BX} = \frac{\boxed{}}{\boxed{}} \text{ (I)}$$

$\triangle BCA \sim \triangle BYX$ $\boxed{}$ test of similarity.

$\therefore \frac{BA}{BX} = \frac{AC}{XY}$ corresponding sides of similar triangles.

$\therefore \frac{\boxed{}}{\boxed{}} = \frac{AC}{9} \therefore AC = \boxed{}$...from (I)

Answer : ACTIVITY: $2AX = 3BX \therefore \frac{AX}{BX} = \frac{3}{2}$

$$\frac{AX + BX}{BX} = \frac{3 + 2}{2} \text{(By Componendo)}$$

$$\frac{AB}{BX} = \frac{5}{3} \text{(I)}$$

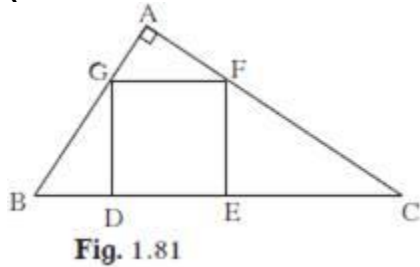
$\triangle BCA \sim \triangle BYX$ (AA test of similarity).

$\therefore \frac{BA}{BX} = \frac{AC}{XY}$ (Corresponding sides of similar triangles).

$\frac{5}{3} = \frac{AC}{9} \therefore AC = 15$ from (I)

Q. 13. In figure 1.81, the vertices of square DEFG are on the sides of $\triangle ABC$, $\angle A = 90^\circ$. Then prove that $DE^2 = BD \times EC$

(Hint : Show that $\triangle GBD$ is similar to $\triangle DFE$. Use $GD = FE = DE$.)



Answer : Proof: In \square DEFG is a square

$$\Rightarrow GF \parallel DE$$

$$\Rightarrow GF \parallel BC$$

Now, In $\triangle AGF$ and $\triangle DBG$

$$\Rightarrow \angle AGF \cong \angle DBG \text{ (corresponding angles)}$$

$$\Rightarrow \angle GDB \cong \angle FAG \text{ (Both are } 90^\circ)$$

$$\Rightarrow \triangle AGF \sim \triangle DBG \dots\dots(1) \text{ (AA similarity)}$$

Now, In $\triangle AGF$ and $\triangle EFC$

$$\Rightarrow \angle AFG \cong \angle ECF \text{ (corresponding angles)}$$

$$\Rightarrow \angle GAF \cong \angle FEC \text{ (Both are } 90^\circ)$$

$$\Rightarrow \triangle AGF \sim \triangle EFC \dots\dots(2) \text{ (AA similarity)}$$

From (1) & (2), we have-

$$\Rightarrow \triangle EFC \sim \triangle DBG$$

$$\Rightarrow \frac{EF}{BD} = \frac{EC}{DG}$$

$$\Rightarrow EF \times DG = BD \times EC$$

Now, \because DEFG is a square

$$\Rightarrow DE = EF = DG$$

$$\Rightarrow DE \times DE = BD \times EC$$

$$\Rightarrow DE^2 = BD \times EC$$